188. Proposed by SAUL EPSTEEN.

Evaluate $I=\int\dfrac{y(y^2+2)dy}{(y^4+3y^2+3)\sqrt{(y^2+1)}}$.

I. Solution by W. W. BEMAN, M. A., Professor of Mathematics in the University of Michigan.

Putting $y^2+1=z^2$ and $z-1/z=u$, the integrand becomes $\dfrac{1}{u^2+3}$; hence integral $=\dfrac{1}{3^{\frac{1}{2}}}\tan^{-1}\dfrac{1}{3^{\frac{1}{2}}}[(1+y^2)^{\frac{1}{2}}-(1+y^2)^{-\frac{1}{2}}]$.

II. Solution by CHRISTIAN HORNUNG, and M. E. GRABER, Tiffin, Ohio.

Let $y^2+1=u^2$, then $I=\int\dfrac{(u^2+1)du}{u^4+u^2+1}=\frac{1}{2}\int\dfrac{du}{u^2+u+1}+\frac{1}{2}\int\dfrac{du}{u^2-u+1}$

$$=\frac{1}{3^{\frac{1}{2}}}\left[\tan^{-1}\frac{2(y^2+1)^{\frac{1}{2}}+1}{3^{\frac{1}{2}}}+\tan^{-1}\frac{2(y^2+1)^{\frac{1}{2}}-1}{3^{\frac{1}{2}}}\right]$$

*** Dr. G. B. M. Zerr obtains the same result and reduces it to the form

$$\frac{1}{\sqrt{3}}\left[\pi+\tan^{-1}-\frac{(3y^2+3)^{\frac{1}{2}}}{y^2}\right].$$

Also solved by J. Scheffer, E. L. Sherwood, H. M. Armstrong, and G. W. Greenwood.
Solved with same result by F. P. Matz.

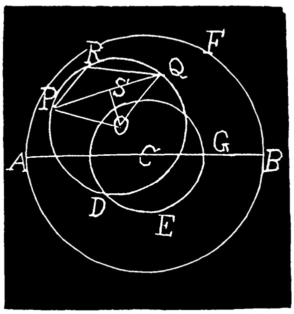
AVERAGE AND PROBABILITY.

130. Proposed by L. C. WALKER, A. M., Santa Barbara, Cal.

Three points are taken at random in a given circle, and a circle passed through them. The probability that the circle through the random points will be wholly in the given circle is $\frac{2}{5}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W Va.

Let PRQ be the triangle formed by joining the three random points P, R, Q in the circle AFB; O the center of the circle circumscribing PRQ. Draw the circle DEG concentric with AFB, both with center C, so that $AD=GB=OP$. Since PQ is known in length and direction, and the angle PRQ is given, if OP is less than AC, the area of DEG represents the number of ways the three points can be taken, so that the circle circumscribing PRQ will lie wholly within AFB.



Let $PQ=2x$, $AC=r$, $\angle POS=\phi$, $\theta=\sin^{-1}(x/r)$, area of segment $PRQ=u$, $\rho=$angle PQ makes with some fixed line.

Then $PO=x\text{cosec}\phi=r\sin\theta\text{cosec}\phi$, $CD=r-x\text{cosec}\phi=r(1-\sin\theta\text{cosec}\phi)$.